Team Contest 4

Team: Ans:

Problem 1. Compute

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}.$$

1 point ☐☐☐

Team: Ans:

Problem 2. Rushil places two non-intersecting ellipses on a plane. Tony then puts 5 points on each of these ellipses. Compute the maximum number of these points that can all lie on the same circle.

1 point ☐☐☐

Team: Ans:

Problem 3. Michelle is learning her 123's. She writes out all the integers from 1 to 1000. What digit appears the most in her list?

2 point ☐☐☐

Team: Ans:

Problem 4. In $\triangle ABC$, let $AB = AC$, and let D and E be on AB and AC, respectively, such that $BD = DE$ and $DE \perp AC$. Compute $\angle EBC$.

2 point ☐☐☐

Team: Ans:

Problem 5. Points $A, B, C, \ldots, Z$ lie on a line in that order such that $AB = 1, BC = 2, \ldots, YZ = 25$. Compute

$$FA \cdot FB \cdot \cdots \cdot FZ.$$

2 point ☐☐☐

Team: Ans:

Problem 6. Say a group of people is *fake* if no three people in the group are all friends. Across all possible fake groups of 2022 people, we can choose n of the people – call them *influencers* – such that every person in the group is friends with at most two of these influencers. Find the maximum possible value of n. Note that friendship is mutual (i.e. if A is friends with B then B is friends with A).

3 point ☐☐☐

Team: Ans:

Problem 7. Compute $a+b+c$ given that a, b, c are real numbers such that

$$a^2+b^2+c^2 = 14,$$
$$a^3+b^3+c^3 = 36,$$
$$a^4+b^4+c^4 = 100.$$

3 point ☐☐☐

Team: Ans:

Problem 8. A permutation $\{x_1, x_2, \ldots, x_8\}$ of $\{1, 2, \ldots, 8\}$ is called *sus* if $x_i < x_{9-i}$ for $i = 1, 2, 3, 4$. Compute the number of sus permutations.

3 point ☐☐☐

Team: Ans:

Problem 9. Deepu is learning his ABC's. Sreeja gives him a string of letters, and his job is to find the number of subsequences of the form `ABC`. For example, the sequence `ABBCC` has 4 such subsequences:

`AB`B`C`C, `AB`B`CC`, `AB`B`C`C, and `A`B`BC`C.

Over all possible strings of 2022 letters, let the maximum number of subsequences `ABC` that Deepu can form be a^b, where a, b are positive integers such that $b > 1$. Compute $a+b$.

2 point ☐☐☐

Team: Ans:

Problem 10. A triple of distinct integers x, y, z taken from the set $\{-3, -2, \ldots, 2, 3\}$ is called *pog* if

$$(x-y)(x^2-y^2)(x^3-y^3) = 3z^3.$$

Compute the number of pog triples.

1 point ☐☐☐